

Estimation of Payload Loads Using Rigid-Body Interface Accelerations

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In the design/analysis process of a payload structural system, the accelerations at the payload/launch vehicle interface obtained from a system analysis using a rigid payload are often used as the input forcing function to the elastic payload to obtain structural design loads. Such an analysis is at best an approximation, since the elastic coupling effects are neglected. This paper develops a method wherein the launch vehicle/rigid payload interface accelerations are modified to account for the payload elasticity. The advantage of the proposed method, which is exact to the extent that the physical system can be described by a truncated set of generalized coordinates, is that the complete design/analysis process can be performed within the organization responsible for the payload design. The proposed method requires the updating of the system normal modes to account for payload changes, but does not require a complete transient solution using the composite system model. The method is applied to a real complex structure, the Viking Spacecraft System. The results obtained by this method for Viking are compared with the exact solution obtained by using the system model.

Introduction

THE design of a structural system is usually governed by its dynamic environments. The design loads are typically obtained from analyses of representative analytical models under the relevant dynamic loadings. This design/analysis process is then iterated until a satisfactory design is achieved. One cycle of this iterative process consists of an update of the responses or load estimation, the associated structural design activity, the revision of the mathematical model, and a repeat of the response analysis. The process is expensive and time consuming, since the mathematical models are large and the delay between the availability of the model and the final dynamic response calculations is substantial.

In aerospace programs it is common to have payloads that are comprised of subsystems which are developed and designed by separate organizations with their respective mathematical models. The design/analysis process involves the integration of these subsystem models to obtain the payload model which is combined with a launch vehicle model. Considerable time and cost are required in the integration of subsystem models and the response analysis, since different computer codes, coordinate systems, and normalization procedures may be used for the subsystem models. In a recent experience, the Viking project, as many as ten organizations were involved in generating various subsystem models and their integration and one design/analysis cycle required up to six months.¹

To reduce the cost and schedule, the payload responses and loads will be obtained by using interface accelerations between the payload and launch vehicle as the input forcing functions to the payload model. The interface accelerations are assumed to be invariant during the subsequent design/analysis iteration.² However, the interface ac-

celerations are estimated from the payload launch vehicle composite model in which the payload and/or its subsystems are represented by their rigid-body inertia properties, since early in the payload program the detailed payload and its subsystems designs are not available. Generally, the resulting payload responses and loads will be conservative, i.e., the response and loads will be greater than the true values, since the elastic coupling effects have been neglected. For certain cases, the responses and loads may become too conservative for the project constraints.

The objective of the present investigation is to develop a method by which the interface accelerations will be modified to include the effects of payload elasticity without having to perform analyses on the launch vehicle/payload composite model. The modification procedure requires the knowledge of payload and launch vehicle/rigid payload characteristics, but the actual integration of the payload model to the physical launch vehicle model and its analysis are not necessary. Thus, the analysis can be performed within the payload organization.

This effort should be especially beneficial to the future shuttle payload design. The payload dynamic load can be obtained by performing a transient analysis of the payload model using interface accelerations as the forcing functions, which are estimated from the shuttle/rigid-body payload model. For certain events, such as the landing of the unpowered shuttle orbiter, the coupling between shuttle orbiter and its payload is critical to both the orbiter and payload loads.³ In these cases, the design loads obtained from the rigid-body payload interface acceleration may be erroneous. The proposed modification includes the payload dynamic characteristics to provide more accurate results. This technique enables the payload organization to update the design loads as the payload is modified during the design/analysis process independent of the shuttle orbiter. A sample problem consisting of the Viking orbiter/lander model will be used to demonstrate the proposed procedure.

Approach

In the present investigation, a method to obtain the launch vehicle/payload interface accelerations from the results of launch vehicle/rigid payload model is developed. The governing equation for a launch vehicle/payload composite model can be written in the finite-element formulation as

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$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \quad (1)$$

where

- $\{x_1\}, \{x_2\}$ = displacement vector representing the launch vehicle and the payload degrees-of-freedom, respectively
 $[M_1], [M_2]$ = mass matrices of the launch vehicle and the payload, respectively
 $[K_{11}]$ = restrained stiffness matrix of the launch vehicle
 $[K_{22}]$ = stiffness matrix of the payload constrained at the interface
 $[K_{21}] = [K_{12}]^T$ = stiffness matrix of the interface structure which connects the payload to the launch vehicle
 $\{F(t)\}$ = external forcing function vector applied to launch vehicle only

Among the launch vehicle degrees-of-freedom, those physically connected to the payload will be defined as the interface degrees-of-freedom $\{x_1\}$. It should be clear now that the matrix $[K_{21}]$ or $[K_{12}]^T$ represents the stiffness between the interface degrees-of-freedom $\{x_1\}$ and the payload degrees-of-freedom $\{x_2\}$. Since $\{x_1\}$ is a subset of the launch vehicle degrees-of-freedom $\{x_1\}$, the following equation can be realized:

$$[K_{21}]\{x_1\} = [K_{12}]^T\{x_1\} = [K_{21}]\{x_1\} = [K_{12}]^T\{x_1\} \quad (2)$$

The motion of the payload will be decomposed into two parts, namely, the elastic motion and the rigid-body motion:

$$\{x_2\} = [\phi_R]\{x_1\} + \{x_e\} \quad (3)$$

The first term on the right-hand side of Eq. (3) is the rigid-body motion in which the matrix $[\phi_R]$ is defined as the motion of the payload due to unit displacements of the interface degrees-of-freedom $\{x_1\}$. Therefore, $[\phi_R]$ is strictly a geometric transformation matrix. The second term $\{x_e\}$ is the elastic motion, or relative motion with reference to the interface. It should be noted that only the elastic motion $\{x_e\}$ will generate loads in the structure. Based on Eq. (3), the following transformation will be made:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \phi_R & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_e \end{Bmatrix} \quad (4)$$

The coefficient matrix on the right-hand side of Eq. (4) is called the transformation matrix. Upon substitution of Eq. (4) into Eq. (1) and pre-multiplying by the transpose of the transformation matrix, the following can be obtained:

$$\begin{bmatrix} M_1 + \phi_R^T M_2 \phi_R & \phi_R^T M_2 \\ M_2 \phi_R & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_e \end{Bmatrix} + \begin{bmatrix} K_{11} + \phi_R^T K_{21} + K_{12} \phi_R + \phi_R^T K_{22} \phi_R & K_{12} + \phi_R^T K_{22} \\ K_{21} + K_{22} \phi_R & K_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_e \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \quad (5)$$

For a rigid payload, it is postulated that the stiffness of the payload is infinitely large, such that no elastic motion of the

payload can be realized, i.e., $\{x_e\} = 0$. Let $\{y\}$ be defined as the displacement vector representing the degrees-of-freedom of such a launch vehicle with rigid payload. Then, according to Eq. (5), the equation of motion can be written as

$$[M_1 + M_{rr}]\{\ddot{y}\} + [K_{11} + \phi_R^T K_{21} + K_{12} \phi_R + \phi_R^T K_{22} \phi_R]\{y\} = \{F(t)\} \quad (6)$$

where

$$[M_{rr}] = [\phi_R]^T [M_2] [\phi_R] = \text{payload rigid-body mass} \quad (7)$$

Physically, $[M_{rr}]$ represents the distribution of rigid payload mass onto the interface degrees-of-freedom. Generally, for a typical payload, nonstructural weights such as instrumentation, electronics, propellants, etc. constitute the major portion of the payload mass, and the weight of the load carrying structure is only a small portion of the total payload mass. Therefore, early in the project the payload rigid-body mass $[M_{rr}]$ can be estimated prior to the actual design since only the mass distribution and geometric configuration are required to establish $[M_2]$ and $[\phi_R]$. It is a common practice that the payload organization will provide an estimated $[M_{rr}]$ to the launch vehicle organization early in the project such that the launch vehicle/rigid-body payload composite model [Eq. (6)] can be constructed. The main purpose of such a model is the verification of the launch vehicle loading. Meanwhile, the interface accelerations of the model can be obtained.

The governing equation for the launch vehicle/rigid payload composite model [Eq. (6)] can be reduced to the generalized coordinate formulation as

$$\{\ddot{V}_i\} + 2[\rho_i][\omega_i]\{\dot{V}_i\} + [\omega_i^2]\{V_i\} = \{G(t)\} \quad (8)$$

where

$$\{y\} = [\phi_i]\{V\} \quad (9a)$$

$$\{G(t)\} = [\phi_i]^T \{F(t)\} \quad (9b)$$

$$[\phi_i]^T [M_1 + M_{rr}] [\phi_i] = [I] = \text{unity matrix} \quad (10a)$$

$$[\phi_i]^T [K_{11} + \phi_R^T K_{21} + K_{12} \phi_R + \phi_R^T K_{22} \phi_R] [\phi_i] = [\omega_i^2] \quad (10b)$$

Clearly, $[\phi_i]$ and $[\omega_i^2]$ are the eigenvectors and eigenvalues of the launch vehicle/rigid payload composite model, respectively. Also it should be noted that modal damping has been included in Eq. (8) and the elements in $[\rho_i]$ represent the percentage of critical damping for each mode.

It is important in the present investigation that the launch vehicle organization has obtained the solutions to the launch vehicle/rigid payload composite model [Eq. (6)] and made them available to the payload organization. Precisely, the following are required: the interface accelerations for the events under consideration $\{y_i\}$, the eigenvalues of the launch vehicle/rigid payload composite system ω_i^2 's, the corresponding eigenvector of the interface degrees-of-freedom $[\phi_i]$ ($[\phi_i]$ is a subset of $[\phi_1]$), and the modal damping $[\rho_i]$. The objective of the present investigation is to use these data provided by the launch vehicle organization to calculate the elastic payload responses and loads under the same dynamic events without having to solve a new launch vehicle/elastic payload composite model similar to Eq. (5).

First, a transformation will be defined as

$$\begin{Bmatrix} x_1 \\ x_e \end{Bmatrix} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \quad (11)$$

where $[\phi_I]$ are the eigenvectors of the launch vehicle/rigid payload system [Eq. (10)] and $[\phi_2]$ are the eigenvectors of the elastic payload constrained at the interface satisfying the following orthogonality conditions:

$$[\phi_2]^T [M_2] [\phi_2] = [I] = \text{unity matrix} \quad (12a)$$

$$[\phi_2]^T [K_{22}] [\phi_2] = [\omega_2^2] = \text{eigenvalues of elastic payload} \quad (12b)$$

Substitution of Eq. (11) into Eq. (5) and pre-multiplying it by the transformation matrix in Eq. (11), one obtains

$$\begin{aligned} & \begin{bmatrix} I & \phi_I^T M_{re} \\ M_{er} \phi_I & I \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} 2\rho_1 \omega_1 & 0 \\ 0 & 2\rho_2 \omega_2 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} \\ & + \begin{bmatrix} \omega_1^2 & \phi_I^T (K_{I2} + \phi_R^T K_{22}) \phi_2 \\ \phi_2^T (K_{21} + K_{22} \phi_R) \phi_I & \omega_2^2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \\ & \times \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} G(t) \\ 0 \end{Bmatrix} \end{aligned} \quad (13)$$

where

$$[M_{er}] = [M_{re}]^T = [\phi_2]^T [M_2] [\phi_R]$$

Again, modal damping has been included in the equation as $[\rho_2]$ representing the percentage of critical damping for each payload mode. Also, in arriving at Eq. (13), the following relationship has been used:

$$[\phi_R] [\phi_I] = [\phi_R] [\phi_I] \quad [K_{21}] [\phi_I] = [K_{21}] [\phi_I] \quad (14)$$

Using the information provided by the launch vehicle organization, namely, $[\phi_I]$, $[\omega_1^2]$, $[\rho_1]$, and the characteristics of the payload such as $[M_{er}]$, $[\omega_2^2]$, $[\phi_2]$, $[\rho_2]$, $[\phi_R]$, and the interface stiffness $[K_{21}]$, the payload organization can construct Eq. (13) except for the generalized external forcing function $\{G(t)\}$. The next step is to define a modal response due to the launch vehicle/rigid payload interface accelerations as

$$\begin{aligned} & \{\ddot{V}_2\} + 2[\rho_2][\omega_2]\{\dot{V}_2\} + [\omega_2^2]\{V_2\} = \\ & -[M_{er}]\{\ddot{y}_I\} = -[M_{er}][\phi_I]\{\ddot{V}_I\} \end{aligned} \quad (15)$$

It should be noted that $\{V_2\}$ can be obtained once the interface accelerations are available. Equations (8) and (15) can now be combined as

$$\begin{aligned} & \begin{bmatrix} I & 0 \\ M_{er} \phi_I & I \end{bmatrix} \begin{Bmatrix} \ddot{V}_I \\ \ddot{V}_2 \end{Bmatrix} + \begin{bmatrix} 2\rho_1 \omega_1 & 0 \\ 0 & 2\rho_2 \omega_2 \end{bmatrix} \begin{Bmatrix} \dot{V}_I \\ \dot{V}_2 \end{Bmatrix} \\ & + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{Bmatrix} V_I \\ V_2 \end{Bmatrix} = \begin{Bmatrix} G(t) \\ 0 \end{Bmatrix} \end{aligned} \quad (16)$$

Then, the solution to Eq. (13) will be decomposed into two parts as

$$\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} V_I \\ V_2 \end{Bmatrix} + \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix} \quad (17)$$

Subtracting Eq. (16) from Eq. (13) and using Eq. (17) one obtains

$$\begin{aligned} & \begin{bmatrix} I & \phi_I^T M_{re} \\ M_{er} \phi_I & I \end{bmatrix} \begin{Bmatrix} \ddot{W}_1 \\ \ddot{W}_2 \end{Bmatrix} + \begin{bmatrix} 2\rho_1 \omega_1 & 0 \\ 0 & 2\rho_2 \omega_2 \end{bmatrix} \begin{Bmatrix} \dot{W}_1 \\ \dot{W}_2 \end{Bmatrix} \\ & + \begin{bmatrix} \omega_1^2 & \phi_I^T (K_{I2} + \phi_R^T K_{22}) \phi_2 \\ \phi_2^T (K_{21} + K_{22} \phi_R) \phi_I & \omega_2^2 \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix} \\ & = \begin{Bmatrix} -\phi_I^T M_{re} \ddot{V}_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} \phi_I^T (K_{I2} + \phi_R^T K_{22}) \phi_2 V_2 \\ \phi_2^T (K_{21} + K_{22} \phi_R) y_I \end{Bmatrix} \end{aligned} \quad (18)$$

Comparing Eqs. (18) and (16), the right-hand side of Eq. (18) is now a function of the interface acceleration $\{\ddot{y}_I\}$ instead of the generalized forcing function $\{G(t)\}$ as in Eq. (16). Consequently, Eq. (18) can be solved within the payload organizations. The structural loads $\{P(t)\}$ can be calculated from the elastic deformation as

$$\{P(t)\} = [S]\{x_e\} = [S][\phi_2](\{V_2\} + \{W_2\}) \quad (19)$$

where $[S]$ is the force coefficients matrix.

The updated interface accelerations which include the effects of payload elasticity are

$$\{\ddot{x}_I\} [\phi_I] \{\ddot{U}\} = [\phi_I] (\{\ddot{V}_I\} + \{\ddot{W}_I\}) = \{\ddot{y}_I\} + [\phi_I] \{\ddot{W}_I\} \quad (20)$$

A Special Case – Statically Determinate Payload

In many cases, the payload is supported on the launch vehicle in such a manner that no internal force will be generated if the payload is undergoing a rigid-body motion with respect to the interface, and the stiffness of the coupled system during such motion is equal to that of the unrestrained launch vehicle (launch vehicle without payload). Such a payload is supported in the statically determinate manner. Mathematically, this implies a certain relationship between the payload stiffness, launch vehicle stiffness, and the interface structural stiffness. This can be derived from the static equilibrium condition of the coupled system:

$$\begin{bmatrix} K_{I1} & K_{I2} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_I \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_I \\ f_2 \end{Bmatrix} \quad (21)$$

If the payload is undergoing a rigid-body motion with respect to the interface degrees-of-freedom, i.e.,

$$\{x_2\} = [\phi_R]\{x_I\} \quad (22)$$

then

$$\{f_2\} = 0 \quad (23)$$

$$\{f_I\} = [K_I]\{x_I\} \quad (24)$$

where $[K_I]$ is the unrestrained launch vehicle stiffness. Substituting Eq. (22) into Eq. (21) and using Eqs. (23), (24), and (2) one obtains

$$\begin{aligned} \{f_2\} &= [K_{21}]\{x_I\} + [K_{22}]\{x_2\} \\ &= ([K_{21}] + [K_{22}][\phi_R])\{x_I\} = 0 \end{aligned} \quad (25a)$$

$$\begin{aligned} \{f_I\} &= [K_{I1}]\{x_I\} + [K_{I2}]\{x_2\} \\ &= ([K_{I1}] + [K_{I2}][\phi_R])\{x_I\} = [K_I]\{x_I\} \end{aligned} \quad (25b)$$

Therefore,

$$[K_{21}] = -[K_{22}][\phi_R] \quad (26a)$$

$$[K_{11}] = [K_I] - [K_{12}][\phi_R] = [K_I] + [\phi_R]^T [K_{22}][\phi_R] \quad (26b)$$

The governing equation for the launch vehicle/rigid payload system, Eq. (16), can be simplified as

$$[M_I + M_{rr}]\{\ddot{y}\}[K_I]\{y\} = \{F(t)\} \quad (27)$$

The corresponding equation for the launch vehicle/elastic payload system can be written as

$$\begin{bmatrix} M_I + M_{rr} & \phi_R^T M_2 \\ M_2 \phi_R & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_I \\ \ddot{x}_e \end{Bmatrix} + \begin{bmatrix} K_I & 0 \\ 0 & K_{22} \end{bmatrix} \begin{Bmatrix} x_I \\ x_e \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \quad (28)$$

In a previous study, the payload loads have been analyzed for the composite system represented by Eq. (28) with the limitation of "small" payloads.⁴ The governing equation for the modification quantities [Eq. (18)] can be greatly simplified as

$$\begin{bmatrix} I & \phi_I^T M_{re} \\ M_{er} \phi_I & I \end{bmatrix} \begin{Bmatrix} \ddot{W}_1 \\ \ddot{W}_2 \end{Bmatrix} + \begin{bmatrix} 2\rho_1 \omega_1 & 0 \\ 0 & 2\rho_2 \omega_2 \end{bmatrix} \begin{Bmatrix} \dot{W}_1 \\ \dot{W}_2 \end{Bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix} = \begin{Bmatrix} -\phi_I^T M_{re} \ddot{V}_2 \\ 0 \end{Bmatrix} \quad (29)$$

It should be noted that few if any payloads are truly supported in a statically determinate manner at launch vehicle/payload interfaces. In most cases, payloads and launch vehicles are coupled through "adapters" whose stiffness rarely satisfies the conditions expressed by Eq. (26). However, for most of the expandable launch vehicles, the so-called "stick models" are used in which the launch vehicle/payload interface is represented by one node with six degrees-of-freedom: three translations and three rotations. Thus, the payload is supported in a statically determinate manner by virtue of the modeling of launch vehicles. Also, one may choose to include the complexity of the adapter in the payload model and to maintain the stick model for the launch vehicle.

Modification of Rigid-Body Mass

As the payload design progresses, the mass distribution and the configuration will change from its original estimation. Consequently, the interface accelerations, eigenvalues, and eigenvectors of the launch vehicle/rigid payload system provided by the launch vehicle organization become erroneous, since the rigid-body mass $[M_{rr}]$ used in the analysis [Eq. (6)] has been modified. A method will be developed here such that the interface accelerations, eigenvalues, and eigenvectors of the launch vehicle/rigid payload system will be updated due to the modification of payload rigid-body mass. Again the advantage is that the updating can be performed within the payload organization without requiring any new information from the launch vehicle organization. For simplicity, a statically determinate payload [Eq. (27)] will be considered. However, the method is equally applicable to the general case [Eq. (6)]. The governing equation for the original system will be rewritten as

$$([M_I] + [M_{rr}])_o \{\ddot{y}\}_o + [K_I]\{y\}_o = \{F(t)\} \quad (30)$$

The subscript o denotes the quantities for the original system. The corresponding equation for the coupled system with modified payload rigid-body mass will be as follows:

$$([M_I] + [M_{rr}])\{\ddot{y}\} + [K_I]\{y\} = \{F(t)\} \quad (31)$$

Comparing the two systems, the mass and stiffness of the launch vehicle, $[M_I]$ and $[K_I]$, respectively, and the external forcing function $\{F(t)\}$ remain unchanged and only the payload rigid-body mass $[M_{rr}]$ has been modified. Generally, modifications of mass distribution and geometric configuration are "small" such that the following assumption can be made:

$$[M_{rr}] = [M_{rr}]_o + [\Delta M_{rr}] \quad (32a)$$

$$\text{Norm}[\Delta M_{rr}] \ll \text{Norm}[M_{rr}]_o \quad (32b)$$

Physically, Eqs. (32) mean that the new payload rigid-body mass $[M_{rr}]$ is equal to the original quantity $[M_{rr}]_o$, plus a modification quantity $[\Delta M_{rr}]$, which is much smaller than the original quantity. This assumption allows the application of matrix perturbation technique for calculating the updated eigenvalues and eigenvectors without performing the complete eigenproblem solution.⁵ Without repeating the derivation, the procedure of obtaining the updated quantities will be outlined as follows:

Let

$$[\phi] = [\phi]_o + [\Delta\phi] \quad (33a)$$

$$[\omega] = [\omega_o] + [\Delta\omega] \quad (33b)$$

where $[\phi]$, $[\omega]$ and $[\phi]_o$, $[\omega_o]$ are the eigenvectors and natural frequencies of the updated [Eq. (31)] and original [Eq. (30)] system, respectively. According to the perturbation theory, their perturbed quantities, $[\Delta\phi]$ and $[\Delta\omega]$ can be obtained from the following⁵:

$$[\alpha] + [\alpha]^T = -[\phi]_o^T [\Delta M_{rr}] [\phi]_o = -[\phi_I]_o^T [\Delta M_{rr}] [\phi_I]_o \quad (34a)$$

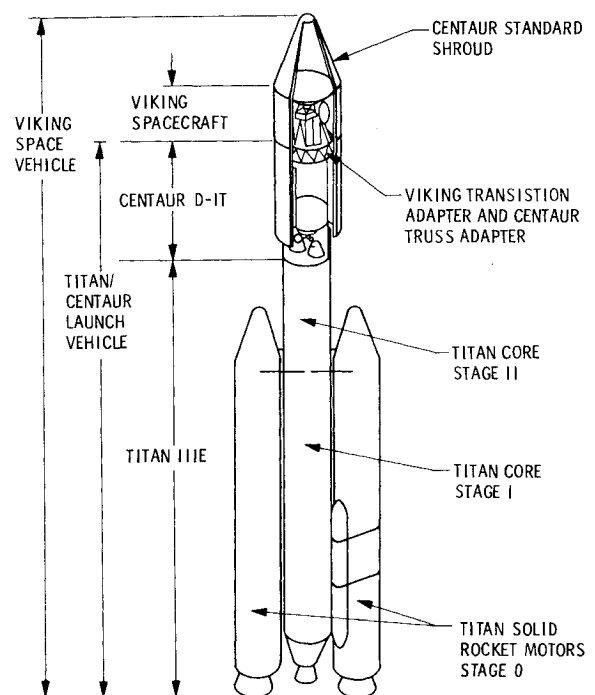


Fig. 1 Viking Titan-Centaur vehicle.

$$2[\omega_o][\Delta\omega] = [\omega_o^2][\alpha] + [\alpha]^T[\omega_o^2] \quad (34b)$$

where

$$[\Delta\phi] = [\phi]_o[\alpha] \quad (35)$$

and $[\phi]_o$ is a subset of $[\phi]_o$, representing the eigenvectors at the interface degrees-of-freedom. Since the quantities at the right-hand side of Eq. (34) are available, the payload organization can obtain the updated eigenvalues and eigenvectors (for the interface degrees-of-freedom only).

Next, the interface accelerations must be updated. By subtracting Eq. (30) from Eq. (31), one obtains

$$([M_I] + [M_{rr}])\{\Delta\ddot{y}\} + [K_I]\{\Delta y\} \equiv -[\Delta M_{rr}]\{\ddot{y}\}_o = -[\Delta M_{rr}]\{\ddot{y}_I\}_o \quad (36)$$

where

$$\{\Delta y\} = \{y\} - \{y_o\} \quad (37)$$

Equation (36), which is obtained by neglecting the second- and higher-order terms of the perturbed quantities such as $[\Delta M_{rr}]\{\Delta\ddot{y}\}$, can be written in the generalized coordinate form as

$$\begin{aligned} \{\Delta\ddot{V}\} + 2[\rho_I][\omega][\Delta\dot{V}] + [\omega^2]\{\Delta V\} = \\ -[\phi]^T[\Delta M_{rr}]\{\ddot{y}_I\}_o = -[\phi_I]^T[\Delta M_{rr}]\{\ddot{y}_I\}_o \end{aligned} \quad (38)$$

where

$$\{\Delta y\} = [\phi]\{\Delta V\} \text{ and } [\phi] = \text{eigenvectors of Eq. (36)} \quad (39)$$

Equation (37) implies that the updated response $\{y\}$ is equal to the original value $\{y\}_o$, plus a modification quantity $\{\Delta y\}$, whose generalized coordinates can be solved by the payload organization using Eq. (38). The updated interface acceleration can be obtained as

$$\{\ddot{y}_I\} = \{\ddot{y}_I\}_o + \{\Delta\ddot{y}_I\} = \{\ddot{y}_I\}_o + [\phi_I]_o([I] + [\alpha])\{\Delta\ddot{V}\} \quad (40)$$

After the interface accelerations of the launch vehicle/rigid payload system have been updated, the loads analysis of the elastic payload as outlined previously can be performed.

Sample Problem

The proposed method was applied to calculated loads and accelerations for a large structural system. A real complex structure was chosen, since the method is exact and little is gained by applying it to a simple system with few degrees-of-freedom other than as a check of the methodology. Such a check was made using a three-degree-of-freedom system; the results were the same as those obtained for the exact solution.

The Viking spacecraft structure was chosen for the sample problem. The Viking spacecraft, two of which have been successfully flown, consists of two subsystems, the Viking Orbiter and the Viking Lander. The structure was designed by loads analyses techniques using modal synthesis methods.^{1,2,6}

The Viking space vehicle system and the Viking spacecraft configurations are shown in Figs. 1 and 2, respectively. To obtain Viking design loads, the space vehicle was subjected to various excitations resulting from flight events such as stage 0 ignition (launch), stage I burnout, and others. Both interfaces, lander/orbiter and spacecraft/launch vehicle, are modeled in a statically determinate fashion.

For purposes of the sample problem, the spacecraft system consisting of the elastic lander and elastic orbiter was chosen to be analyzed. The orbiter was represented by 62 normal

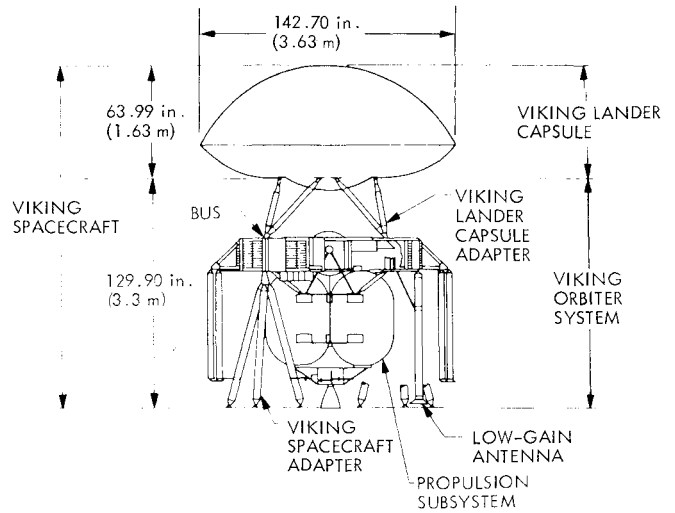


Fig. 2 Viking spacecraft.

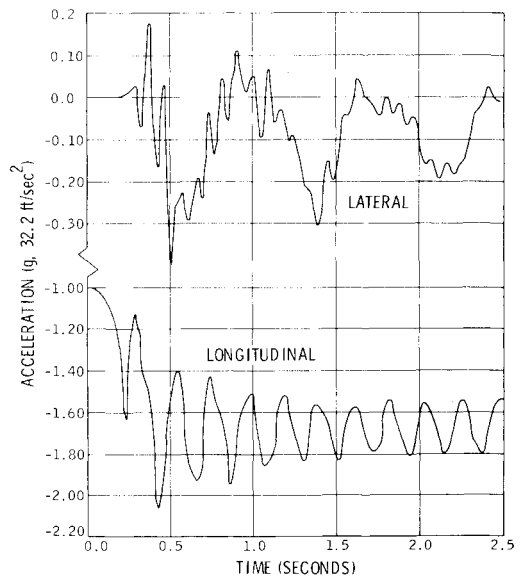


Fig. 3 Viking spacecraft base acceleration for stage 0 ignition.

modes covering the range up to 70 Hz; 60 normal modes for the lander were used to cover a comparable frequency range. To obtain meaningful member loads, system normal modes up to 40 Hz were required. Modes for the spacecraft cantilevered at the spacecraft/launch vehicle interface were obtained by modal synthesis, for both an elastic lander and a rigid lander. The input forcing function consists of the six-degree-of-freedom acceleration at the base of the spacecraft for the stage 0 ignition event. This acceleration time history, shown in Fig. 3, was obtained from the launch vehicle system analysis.

The process for obtaining member loads and accelerations by the proposed method was as follows:

1) Using the orbiter model with rigid lander the lander/orbiter interface acceleration and the Viking Lander Capsule Adapter (VLCA) truss member forces were calculated using Eq. (27). In this solution 42 normal modes up to 46 Hz were used.

2) The interface acceleration from step 1 was now used to obtain the time histories of the lander generalized coordinates, V_2 of Eq. (15). The lander was represented by 48 normal modes up to 60 Hz.

3) The lander generalized coordinates of step 2 were used to obtain the elastic effect on the member loads and ac-

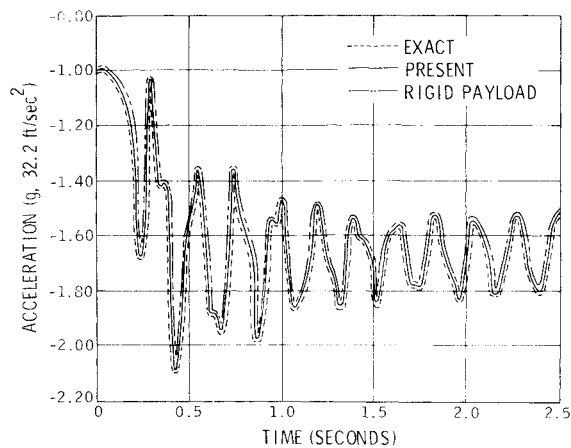


Fig. 4 Viking lander/orbiter interface acceleration in longitudinal direction at stage 0 ignition.

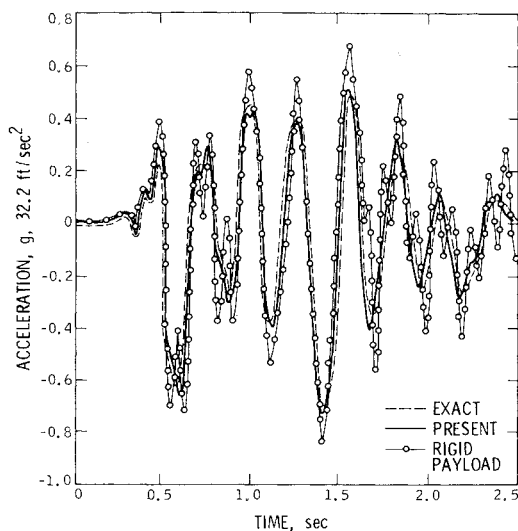


Fig. 5 Viking lander/orbiter interface acceleration in lateral direction at stage 0 ignition.

celerations of the loading by solving Eq. (29) with a total of 78 normal modes up to 48 Hz. Equation (29) was diagonalized to obtain this solution.

4) The results of steps 1 and 3 were added to obtain the solution as prescribed by Eq. (17).

5) The exact solution was generated for comparison by applying the same forcing function to the composite lander/orbiter model using 78 normal modes.

Figures 4 and 5 show a comparison of three time histories for the interface accelerations: the exact solution, the solutions obtained by the proposed method, and the solution by using a rigid rather than a flexible lander. Figures 6 and 7 show similar data for selected VLCA members. The experiences in applying the method to the sample problem can be summarized as follows:

1) **Modal Truncation:** It is important that all significant modes be retained in the equations. This is typically done by using a frequency and effective mass^{7,8} criteria. Extreme modal truncation can give erroneous results. The proposed method is exact only to the extent to which the generalized coordinates describe the physical system.

2) **Modal Damping:** Care should be taken in using proper damping values. The values used for the subsystem modes in the sample problem were derived from test data. In generating modal damping on system level, engineering judgment must be applied. The triple matrix product on the composite level is

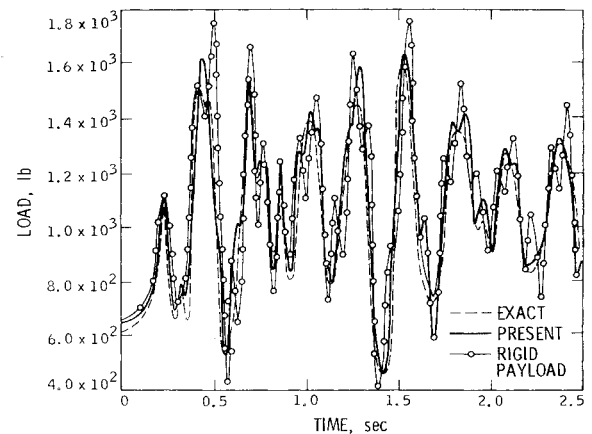


Fig. 6 Viking lander capsule adapter member 750 loads at stage 0 ignition.

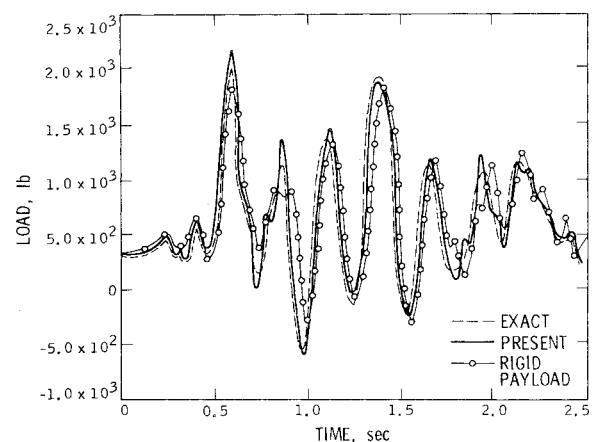


Fig. 7 Viking lander capsule adapter member 754 loads at stage 0 ignition.

used as a basis for making these judgments in combination with mode identification and energy distribution. In the past this approach has been successfully used and has been verified by test.⁹ It has been established in the sample problem that the solution is sensitive to damping.

3) **Initial Conditions:** The initial conditions on the generalized or modal coordinate level are required in the solutions of Eqs. (27) and (15). This can easily be overlooked and will result in erroneous answers.

Examination of the results shows that for the stage 0 ignition event there is no elastic response of the spacecraft system in the longitudinal direction. A comparison of Figs. 3 and 4 shows that the spacecraft system acts as a rigid body. Since the spacecraft longitudinal modes have a higher frequency than the excitation, the difference between the three time histories is negligible.

Figure 5 shows that the proposed method does improve the lateral acceleration time history at the interface. In all cases the peak acceleration calculated by the proposed method is closer to the exact solution than the rigid-body approximation. A comparison of the member force time histories shows that the proposed method gives generally better results than the member forces calculated from the rigid-body acceleration.

Since it has been mathematically established that the method is exact provided that the formulation represents the physical system, the discrepancy between the exact solution and the proposed method can only be explained by the following shortcomings in the analysis: 1) truncation of normal modes, and 2) the treatment of damping. The sample

problem has shown that it is very important that special attention be paid in both of these areas to obtain good results.

Concluding Remarks

A method has been developed by which a transient loads analysis can be performed within the payload organization. The method requires certain information from the launch vehicle organization including the launch vehicle/rigid payload composite frequencies, modal damping, modal displacements for the interface degrees-of-freedom, and interface responses due to various events. The advantages of the method are the ability of the payload organization to perform a complete design/load analyses cycle independently, which eliminates the costly and time consuming interfaces between the launch vehicle and payload organizations. The presently developed method has no limitation regarding the relative size of the payload to launch vehicle or the type of forcing function used in the transient analysis.

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